

# Propagation of spatially entangled qudits through free space

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We show the propagation of entangled states of high-dimensional quantum systems. The qudits states were generated using the transverse correlation of the twin photons produced by spontaneous parametric down-conversion. Their free-space distribution was performed at the laboratory scale and the propagated states maintained a high-fidelity with their original form. The use of entangled qudits allow an increase in the quantity of information that can be transmitted and may also guarantee more privacy for communicating parties. Therefore, studies about propagating entangled states of qudits are important for the effort of building quantum communication networks.

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## I. INTRODUCTION

Most of the applications in quantum communication, like teleportation [1] and quantum cryptography [2], rely on the assumption that the communicating parties are capable to transmit entangled particles between themselves. Because of the practical potential that an implementation of these applications over distant locations could have, the propagation of entangled states of qubits have been theme of recent studies. The first remarkable work used optical-fibers links to send energy-time entangled qubits for receivers separated by more than 10 Km [3]. A test of Bell inequality [4] showed that the two-photon state was still an entangled state and it was the first evidence that quantum correlations could be maintained over significant distances. Another interesting work was about the free-space distribution of polarization entangled qubits through the atmosphere [5]. As it was emphasized in that paper, "... one of the benefits of a free-space distribution of quantum entanglement is the possibility of bridging large distances by additional use of space infrastructure...". Observers were separated by 600 m and the quality of the entanglement of the propagated state had been guaranteed by a violation of Bell's inequality by more than four standard deviations.

Even though promising new experiments have had success propagating entangled qubits over farther distances [6, 7], it has been demonstrated theoretically by E. Waks *et al.* [8], that due to channel losses and dark counts, the communication length cannot surpass the order of 100 km while using entangled photons and joint measurements. For this reason we believe that the control of the technique to create and transmit entangled photons lying in a  $D$ -dimensional ( $D \geq 3$ ) Hilbert space, will be a crucial step in the near future. They allow an increase in the quantity of information that can be transmitted

per pair shared and will then require less effort of quantum repeaters when transmitting information in a global scale. Another advantage is that the use of entangled qudits may increase the security of the entanglement based quantum cryptography protocols against certain types of eavesdropping attacks [9].

In this work, we report the experimental free-space propagation of two entangled 4-dimensional qudits or ququarts. Following the studies developed at references [10, 11, 12], the ququarts entangled state was generated by using the transverse spatial correlation of the photon pairs (*biphotons*) produced by spontaneous parametric down-conversion (SPDC) and two four-slits sets, where they were transmitted through. The propagation was performed at laboratory scale and the propagated state observed had a high-fidelity with its original form. The presence of interference when the two photons are detected in coincidence is used as an experimental measurement for showing that the state of the propagated ququarts is entangled and an evidence of the good quality of the entanglement is discussed.

## II. THEORY

In Ref. [10], it was showed that the state of the biphotons when they are transmitted through generic apertures is

$$|\Psi\rangle = \int dq_1 \int dq_2 \mathcal{F}(q_1, q_2) |1q_1\rangle |1q_2\rangle, \quad (1)$$

with the biphoton amplitude given by

$$\begin{aligned} \mathcal{F}(q_1, q_2) \propto & \int dx_1 \int dx_2 e^{i\frac{k}{8z_A}(x_2-x_1)^2} e^{-i(q_1x_1+q_2x_2)} \\ & \times A_1(x_1)A_2(x_2)W(\tfrac{1}{2}(x_1+x_2);z_A), \end{aligned} \quad (2)$$

where  $q_j$  and  $x_j$  are the wave vector and position transverse components, respectively, of the down-converted

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photons in modes  $j = 1, 2$ .  $A_j(x_j)$  is the transmission function of the aperture in mode  $j$  and  $W(\xi; z_A)$  is the pump beam transverse field profile at the plane of the apertures ( $z = z_A$ ).

We consider the apertures where the twin photons are sent through as two identical four-slits. The separation between two consecutive slits is  $d$  and  $a$  is the slits half width. If the pump beam transverse profile,  $W(\xi; z_A)$ , is non-vanishing only at a small region of space, it can be experimentally demonstrated that Eq. (1) becomes [11]

$$|\Psi\rangle = \frac{1}{2} \sum_{l=-\frac{3}{2}}^{\frac{3}{2}} e^{ik\frac{d^2 l^2}{2z_A}} |l\rangle_1 \otimes |-l\rangle_2, \quad (3)$$

where

$$|l\rangle_j \equiv \sqrt{\frac{a}{\pi}} \int dq_j e^{-iq_j l d} \text{sinc}(q_j a) |1q_j\rangle. \quad (4)$$

The  $\{|l\rangle_j\}$  states represent the photon in mode  $j$  transmitted by the slit  $l$  and satisfy  ${}_j\langle l|l'\rangle_j = \delta_{ll'}$ . The two-photon state in Eq. (3) is a maximally entangled state of two ququarts, where we can see that, when the photon in mode 1 is transmitted by the slit  $l$  the photon in mode 2 will pass through the symmetrically opposite slit  $-l$ .

Now we want to show that the state of Eq. (3) can be propagated at the free-space. The biphotons propagation will be through two independent channels which have distinctly lenses with focal length  $f_1$  and  $f_2$  (Fig. (1)). These lenses are placed at a distance  $z_{L_i}$  from their respective apertures. We calculated the two-photon state transmitted by generic apertures and propagated through these channels to the planes of the image formation ( $z_{I_i}$ ).

For simplicity, the conditions used for image formation are  $z_{I_i} - z_{L_i} = z_{L_i} - z_A = 2f_i$ . To obtain the image state, a general form for it must be assumed

$$|\Psi\rangle_{Im} = \int dq_1 \int dq_2 \mathcal{I}(q_1, q_2) |1q_1\rangle |1q_2\rangle. \quad (5)$$

Calculating the amplitude of the coincidence detection [13] of the biphotons at the planes of the image formation using two different methods, we could establish the form of  $\mathcal{I}(q_1, q_2)$ . The first amplitude's calculus was done considering the state of Eq. (1) and the electric-field operators describing the evolution of the photons through their channels. The second method used the state of Eq. (5) and the expression for the electric-field operator at the point of image formation. Matching their results we obtained

$$\begin{aligned} \mathcal{I}(q_1, q_2) &\propto \int dq'_1 \int dq'_2 \mathcal{F}(q'_1, q'_2) \\ &\times e^{-if_1(q_1+q'_1)^2/2k} e^{-if_2(q_2+q'_2)^2/2k}. \end{aligned} \quad (6)$$

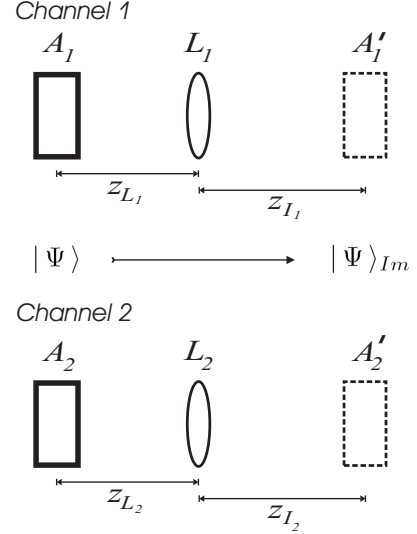


FIG. 1: Channels for the free-space propagation.  $A_1$  and  $A_2$  are generic apertures.  $L_1$  and  $L_2$  are convergent lenses with focal lengths  $f_1$  and  $f_2$ , respectively.  $A'_1$  and  $A'_2$  are the images of the apertures.

When the apertures from which the twin photons were transmitted by are two identical four-slits, described above, the function  $\mathcal{F}(q_1, q_2)$  will be given by [10]

$$\mathcal{F}(q_1, q_2) \propto \sum_{l=-\frac{3}{2}}^{\frac{3}{2}} e^{ik\frac{d^2 l^2}{2z_A}} e^{-iq_1 l d} \text{sinc}(q_1 a) e^{iq_2 l d} \text{sinc}(q_2 a). \quad (7)$$

Thus, inserting Eq. (7) into Eqs. (6) and (5) will give the state of the propagated ququarts

$$|\Psi\rangle_{Im} = \frac{1}{2} \sum_{l=-\frac{3}{2}}^{\frac{3}{2}} e^{ik\frac{d^2 l^2}{2z_A}} |-l\rangle_1 \otimes |l\rangle_2, \quad (8)$$

which has the same form of the two-photon state at the plane of the four-slits, Eq. (3), showing that this state can be propagated at the free-space. A more important result can be obtained when one uses Eq. (5) and Eq. (6) to show that the general state for the twin-photons after being transmitted through generic apertures (See Eq. (1)), will always be reconstructed at the planes of their images. It is not worthless to mention that the theory doesn't require the use of identical lenses what means that the receivers of the entangled qudits can be at different distances from the apertures (source).

### III. EXPERIMENTAL SETUP AND RESULTS

The setup represented in Fig. 2 was carried out to experimentally demonstrate the free-space propagation of the ququarts entangled state described by Eq. (3). A 5-mm-long  $\beta$ -barium borate crystal is used to generate type-II parametric down-conversion luminescence when pumped by a 100 mW pulsed laser beam. Down-converted photons with the same wavelength ( $\lambda = 826$  nm) are selected using interference filters. Two identical four-slits ( $A_1$  and  $A_2$ ) are placed at their exit path at the same distance  $z_A = 200$  mm from the crystal ( $z = 0$ ). The slit width is  $2a \approx 0.09$  mm and the distance between two consecutive slits is  $d \approx 0.17$  mm. To guarantee that the function  $W(\xi; z_A)$  is non-vanishing only at a small region of the space the pump beam transverse profile was focused at the plane of these apertures. After being transmitted by the four-slits the biphotons are propagated at the free-space and through two identical lenses ( $L_1$  and  $L_2$ ), with focal length of  $f = 150$  mm and placed at a distance of  $z_L = 500$  mm from the crystal. At the plane of the image formation of the apertures ( $z_I = 800$  mm), avalanche photodiode detectors ( $D_1$  and  $D_2$ ) are placed. Single and coincidence counts are measured by the detectors and in front of each detector there is a single slit of width 0.1 mm (parallel to the four-slits).

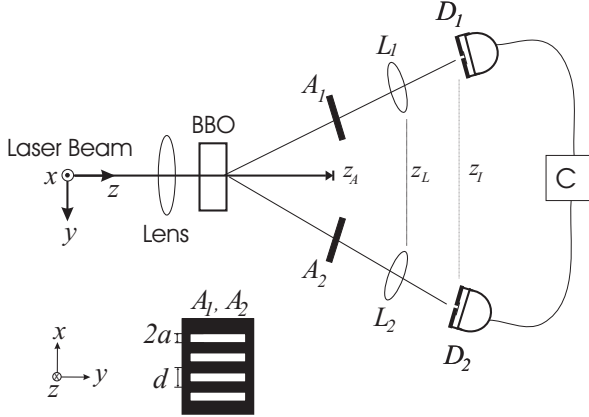


FIG. 2: Outline of the experimental setup.  $A_1$  and  $A_2$  are four-slit apertures,  $L_j$  lens,  $D_j$  a detector and C is a coincidence counter.

Coincidence selective measurements onto the basis  $\{|l\rangle_1|l'\rangle_2\}$  are performed to determine the two-photon image state. Detector  $D_1$  is fixed at a region in space where the image of slit  $l$  of the four-slit in channel 1 is formed while detector  $D_2$  is scanning, in the  $x$  direction, the entire region where the image of the other four-slit is formed. Four measurements of this kind, with detector  $D_1$  going from the image of the slit for which  $l = -\frac{3}{2}$  to the image of the slit with  $l = \frac{3}{2}$ , will determine the probability amplitudes in the sixteen basis states  $\{|l\rangle_1|l'\rangle_2\}$ .

If the theoretical result of Eq. (8) for the state of the twin photons at the plane of image formation is cor-

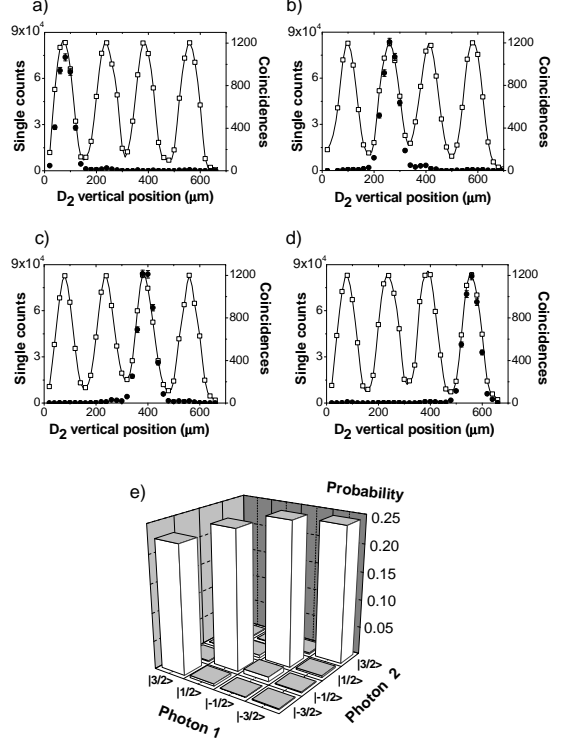


FIG. 3:  $D_2$  single counts ( $\square$ ) and coincidence counts ( $\bullet$ ) measured in 20 s, simultaneously, with  $D_1$  fixed behind the image of the slit  $l$  in channel 1 and  $D_2$  scanning in  $x$  direction the image of the four-slit in channel 2. From left to right the single count peaks are the slits  $l' = -\frac{3}{2}, \dots, +\frac{3}{2}$ .  $D_1$  is fixed behind the slit  $l$  (a)  $+\frac{3}{2}$ , (b)  $+\frac{1}{2}$ , (c)  $-\frac{1}{2}$  and (d)  $-\frac{3}{2}$ . (e) 3D-Histogram of the probabilities measured for all basis states  $\{|l\rangle_1|l'\rangle_2\}$ .

rect, coincidences peaks will occur only when detector  $D_2$  passes by the image of slit for which  $l' = -l$ . However, the classical correlated state given by

$$\rho_{cc} = \frac{1}{2} \sum_{l=-\frac{3}{2}}^{\frac{3}{2}} |l\rangle_1\langle l| \otimes |-l\rangle_2\langle -l|, \quad (9)$$

predicts the same experimental result. And then to guarantee that the image state is indeed given by a coherent superposition (Eq. (8)), the detectors are moved to a distance of 200 mm from the image formation plane and conditional fourth-order interference patterns [14, 15] are measured. As it was demonstrated in Ref. [11], when we treat the spatial correlations of two photons, the observation of a fourth-order interference pattern with conditional fringes is a sufficient signature for entanglement. If the correlations between the propagated ququarts were classical, the coincidence count rate, at this new configuration of the setup, would exhibit only a single slit diffraction pattern.

The experimental data recorded at the plane of image formation of the four-slits is showed in Fig. 3. We can see that the results are in agreement with Eq. (8) because coincidences peaks were only observed when  $D_2$  was scanning the image of the slit symmetrically opposite to that which detector  $D_1$  was fixed. Figure 3(e) is a histogram constructed using all the coincidences recorded in the four measurements performed. The probability related is the chance for the propagated ququarts state, selected in coincidence, be in the form of one of the basis states. The fact that the probabilities for the states  $|l\rangle_1|l\rangle_2$  are almost equally weighted and all the others probabilities null is a strong evidence that the image state is a *maximally* entangled state of ququarts. This means that the states  $|l\rangle_1|l\rangle_2$  will have almost the same amplitudes at the coherent superposition of the obtained image state (See Eq. (10)).

The fourth-order interference patterns measured when the detectors were moved to a distance of 200 mm from the image formation plane and the propagated ququarts detected in coincidence are showed in Fig. 4. Coincidence measurements were made as a function of the  $x$  position of the detector  $D_1$  while  $D_2$  was kept fixed. The results are shown in Fig. 4: (a)  $D_2$  fixed at  $x_2 = 0$  mm; (b)  $D_2$  fixed at  $x_2 = 0.6$  mm. The visibilities of the interference patterns are  $v_a = 0.86 \pm 0.05$  and  $v_b = 0.83 \pm 0.04$ , respectively. One can easily observe the conditionality of the fringes of these patterns. As mentioned before, the presence of conditional interference patterns, at this new configuration of the setup, demonstrates that the image state is pure and entangled.

The propagated ququarts state obtained from the measurements can then be written as:

$$|\Psi\rangle_{Im} = 0.49 \left| -\frac{1}{2}, +\frac{1}{2} \right\rangle + 0.50 \left| +\frac{1}{2}, -\frac{1}{2} \right\rangle + e^{i\frac{kd^2}{z_A}} (0.47 \left| -\frac{3}{2}, +\frac{3}{2} \right\rangle + 0.49 \left| +\frac{3}{2}, -\frac{3}{2} \right\rangle), \quad (10)$$

which has a fidelity of  $F = 0.98 \pm 0.06$  to the original state of the ququarts given by Eq. (3). This prove that we were able to propagate the entangled ququarts state keeping a high-fidelity to its original form. The phase in Eq. (10) was not measured because it can be cancelled out by choosing right values for  $d$  and  $z_A$  or by adding an appropriate external phase to a given slit.

#### IV. DISCUSSION AND CONCLUSION

We believe that the process of entangled qudits propagation, described above, can be implemented for larger distances. As it was demonstrated, at the plane of image formation of the four-slits, the ququarts entangled state (Eq. (3)) is reconstructed with high-fidelity. Besides this, it is well know that different configurations of lenses can be used after objects to make their image appear at long distances. So, one can see that the use of such configuration after the apertures would allow the

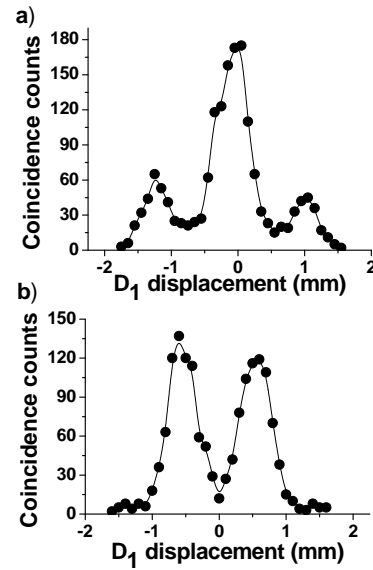


FIG. 4: Fourth-order interference patterns as a function of  $D_1$  position. They were recorded when the propagated photons were detected in coincidence at a distance of 200 mm of the image formation plane. (a)  $D_2$  is kept fixed at  $x_2 = 0$  mm. (b)  $D_2$  is kept fixed at  $x_2 = 0.6$  mm. The solid curve is a guide to the reader's eyes.

transmission of the entangled photons through more significant distances. In Ref. [5], two telescopes were used to propagate entangled qubits over more than 500 m.

In conclusion, we have presented a principle to propagate entangled states of qudits, generated using the transverse correlation of the twin photons produced by SPDC, at the free-space. Up to our knowledge this is the first report of a propagation of entangled states of high-dimensional quantum systems. The experimental test performed obtained a propagated state with a high-fidelity to its original form. The benefits of a free-space distribution of quantum entanglement were already discussed at Ref. [5]. The advantages of using entangled states of high-dimensions quantum systems to transmit information come both from the increase at the quantity of information that can be encoded at the entangled quantum systems and for the possibility of performing more safers quantum cryptography protocols. For these reasons we believe that the work presented in this paper is an important step that can be considered in the effort of building quantum communication networks.

#### V. ACKNOWLEDGMENTS

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